Salvador Macías Hernández
717320

PROYECTO DE MATEMÁTICAS AVANZADAS

Enfriamiento de un Cilindro

Condiciones

$T_0 = 100\ ^{\circ}C$

$a = 1\ m$

$k = 8$

$c = 2\ m^2 / s$

$$\Delta T - \frac{1}{c}\frac{\partial T}{\partial t} = 0$$

$$\frac{\partial T}{\partial r}\Big|_{r=0} = -kT\Big|_{r=a}$$

$$\Delta T - \frac{1}{c}\frac{\partial T}{\partial t} = \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) - \frac{1}{c}\frac{\partial T}{\partial r} = 0$$

Considerando las variables
que intervienen en la temperatura se tiene que :
$T(r, \emptyset, z, t)$ **dende** ϕ **y z no intervienen**
Entonces $T(r, t) = R(r)\,T(t)$
Sustituyendo en la ecuación original .

$$\left(\frac{\partial^2 R}{\partial r^2} T + \frac{1}{r}\frac{\partial R}{\partial r} T\right) - \frac{1}{c}\frac{\partial T}{\partial t} R = 0$$

$$T\left(\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r}\right) = \frac{1}{c}\frac{\partial T}{\partial t} R$$

$$\left(\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r}\right)\frac{1}{R} = \frac{1}{c}\frac{\partial T}{\partial t}\frac{1}{T} = -\lambda$$

Dividiendo en dos ecuaciones se tiene :

$$\left(\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r}\right)\frac{1}{R} = -\lambda \qquad \frac{1}{c}\frac{\partial T}{\partial t}\frac{1}{T} = -\lambda$$

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r} + \lambda R = 0 \qquad \frac{1}{c}\frac{\partial T}{\partial t} + \lambda T = 0$$

$$r\frac{\partial^2 R}{\partial r^2} + \frac{\partial R}{\partial r} + r\lambda R = 0 \qquad \frac{\partial T}{\partial t} + c\lambda T = 0$$

De estas ecuaciones se sabe que la solucion tiene la forma :

$$x^2 \frac{\partial^2 R}{\partial r^2} + x \frac{\partial R}{\partial r} + (x^2 - \nu) R = 0$$

si $\nu = 0$, se tiene :

$$x^2 \frac{\partial^2 R}{\partial r^2} + x \frac{\partial R}{\partial r} + (x^2 - 0) R = 0$$

Esta ecuación tiene soluciones independientes

$R = J_0 (x)$

$R = N_0 (x) \quad N_0 (x) \to \infty$, Si $x \to 0$

La solución es acotada en $x = 0$ y $r = 0$

$R = J_0 \left(r \sqrt{\lambda}\right)$

Por otra lado la ecuacion :

$$\frac{\partial T}{\partial t} + c\lambda T = 0$$

Tiene solución de la siguiente forma :

$$T = Ae^{-\lambda c t}$$

Uniendo las dos ecuaciones tenemos :

$$T = A J_0 \left(r \sqrt{\lambda}\right) e^{-\lambda c t}$$

Aplicando las condiciones de frontera

$$\frac{\partial T}{\partial r} \Big|_{r=0} = -kT \Big|_{r=1}$$

con $a = 1$, $k = 8$

$T = A J_0 \left(r \sqrt{\lambda}\right) e^{-\lambda c t}$

Se nececita igular la derivada de T en el centro con la temperatura en el contorno de la barra

$$\frac{\partial}{\partial x} [x^{\nu} J_{v} (x)] = x^{\nu} J_{v-1} (x);$$

De la ecuación de Bessel $v = 0$

Sustituyendo tenemos

$$\frac{\partial}{\partial x} [J_0 (x)] = - J_{-1} (x)$$

Sustituyendo en la ecuación de frontera :

$$\frac{\partial T}{\partial r} \Big|_{r=1} = - Ae^{-\lambda c t} \sqrt{\lambda}\, J_1 \left(r \sqrt{\lambda}\right)$$

$$\frac{\partial T}{\partial r} \Big|_{r=1} = - Ae^{-\lambda c t} \sqrt{\lambda}\, J_1 \left(\sqrt{\lambda}\right)$$

$$- T \Big|_{r=1} = - Ae^{-\lambda c t} J_0 \left(\sqrt{\lambda}\right)$$

Igualando terminos tenemos :

$$\frac{\partial T}{\partial r} \Big|_{r=1} = - KT \Big|_{r=1}$$

$$- Ae^{-\lambda c t} \sqrt{\lambda}\, J_1 \left(\sqrt{\lambda}\right) = - Ae^{-\lambda c t} J_0 \left(\sqrt{\lambda}\right)$$

$$\sqrt{\lambda}\, J_1 \left(\sqrt{\lambda}\right) = J_0 \left(\sqrt{\lambda}\right) = \lambda$$

Ahora si

$\sqrt{\lambda} = x$

$$F(x) = \frac{X J_1(X)}{J_0(X)} - K = 0$$

donde K =

$$X J_1(X) + J_0(X) = 0$$

Calculando con Matemática las raices tenemos :

```
<< NumericalMath`BesselZeros`
```

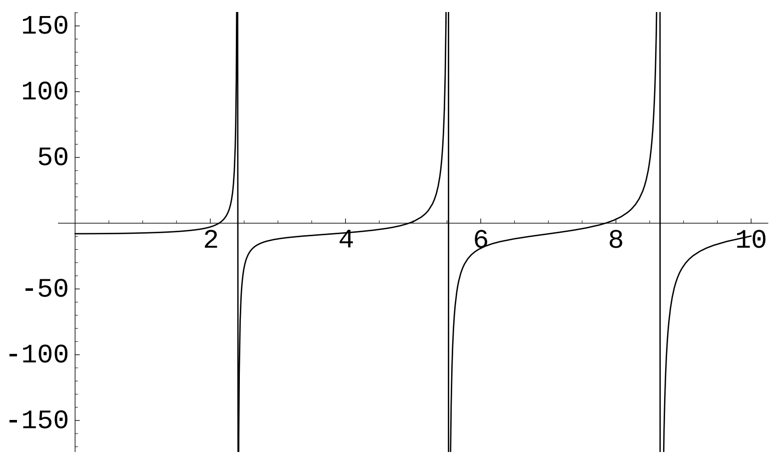
```
k = 8
```

```
8
```

$$F = \frac{x\,\text{BesselJ}[1, x]}{\text{BesselJ}[0, x]} - k$$

$$-8 + \frac{x\,\text{BesselJ}[1, x]}{\text{BesselJ}[0, x]}$$

```
Plot[F, {x, 0, 10}]
```




```
- Graphics -
```

```
Table[N[FindRoot[F, {x, i*2}]], {i, 50}]

{{x → 2.12864}, {x → 4.93838}, {x → 7.84636}, {x → 7.84636},
 {x → 10.8271}, {x → 13.8566}, {x → 13.8566}, {x → 16.9179},
 {x → 16.9179}, {x → 19.9999}, {x → 23.0959}, {x → 23.0959},
 {x → 26.2017}, {x → 29.3144}, {x → 29.3144}, {x → 32.4324},
 {x → 35.5543}, {x → 35.5543}, {x → 38.6793}, {x → 38.6793},
 {x → 41.8066}, {x → 44.9358}, {x → 44.9358}, {x → 48.0667},
 {x → 51.1988}, {x → 51.1988}, {x → 54.3319}, {x → 57.466},
 {x → 57.466}, {x → 60.6009}, {x → 60.6009}, {x → 63.7363},
 {x → 66.8724}, {x → 66.8724}, {x → 70.0089}, {x → 73.1459},
 {x → 73.1459}, {x → 76.2833}, {x → 79.4209}, {x → 79.4209},
 {x → 82.5589}, {x → 82.5589}, {x → 85.6972}, {x → 88.8356},
 {x → 88.8356}, {x → 91.9743}, {x → 95.1132}, {x → 95.1132},
 {x → 98.2522}, {x → 101.391}}

Lamda = {2.12863854824789866`,
   4.93837900369304083`, 7.84635805855231627`,
   10.8270594840219057`, 13.8566370541018679`,
   16.9178804612445565`, 19.9998799671092868`,
   23.0958847519174259`, 26.2016518303453915`,
   29.3144371196545314`, 32.4324098023631091`,
   35.5543113620956763`, 38.6792527401472909`,
   41.8065903027003216`, 44.9358478125526428`,
   48.066665987567827`, 51.1987690484806368`,
   54.3319420146466125`, 57.4660149644644047`,
   60.600851889573768`, 63.7363427043608687`,
   66.8723973928784509`, 70.0089416421344168`,
   73.1459135872644061`, 76.2832613205322296`,
   79.4209409733928772`, 82.5589152241589552`,
   85.6971521256303603`, 88.8356241711872307`,
   91.9743075581179425`, 95.1131815782980005`,
   98.2522281393728391`, 101.391431360943218`}^
  2

{4.5311, 24.3876, 61.5653, 117.225, 192.006, 286.215, 399.995,
 533.42, 686.527, 859.336, 1051.86, 1264.11, 1496.08, 1747.79,
 2019.23, 2310.4, 2621.31, 2951.96, 3302.34, 3672.46, 4062.32,
 4471.92, 4901.25, 5350.32, 5819.14, 6307.69, 6815.97, 7344.,
 7891.77, 8459.27, 9046.52, 9653.5, 10280.2}
```

$$Ai = \frac{100 \int_0^1 BesselJ[0, r * X] * r \, dr}{\int_0^1 BesselJ[0, r * X] * r \, dr}$$

— *Integrate::gener : Unable to check convergence*

— *Integrate::gener : Unable to check convergence*

100

$$100 \int_0^1 BesselJ\left[0, r * \sqrt{\lambda}\right] * r \, dr$$

$$\frac{100\, BesselJ\left[1, \sqrt{\lambda}\right]}{\sqrt{\lambda}}$$

$$\int_0^1 BesselJ\left[0, r * \sqrt{\lambda}\right]^2 * r \, dr$$

$$\frac{1}{2} \left(BesselJ\left[0, \sqrt{\lambda}\right]^2 - BesselJ\left[-1, \sqrt{\lambda}\right] BesselJ\left[1, \sqrt{\lambda}\right]\right)$$

$$\frac{1}{2} \left(BesselJ\left[0, \sqrt{\lambda}\right]^2 + BesselJ\left[1, \sqrt{\lambda}\right]^2\right)$$

$$Ai = \frac{\frac{100\, BesselJ\left[1, \sqrt{Lamda[[n]]}\right]}{\sqrt{Lamda[[n]]}}}{\frac{1}{2} \left(BesselJ\left[0, \sqrt{Lamda[[n]]}\right]^2 + BesselJ\left[1, \sqrt{Lamda[[n]]}\right]^2\right)}$$

— *Part::pspec : Part specification n is neither an integer nor a list of integers.*

— *Part::pspec : Part specification n is neither an integer nor a list of integers.*

— *Part::pspec : Part specification n is neither an integer nor a list of integers.*

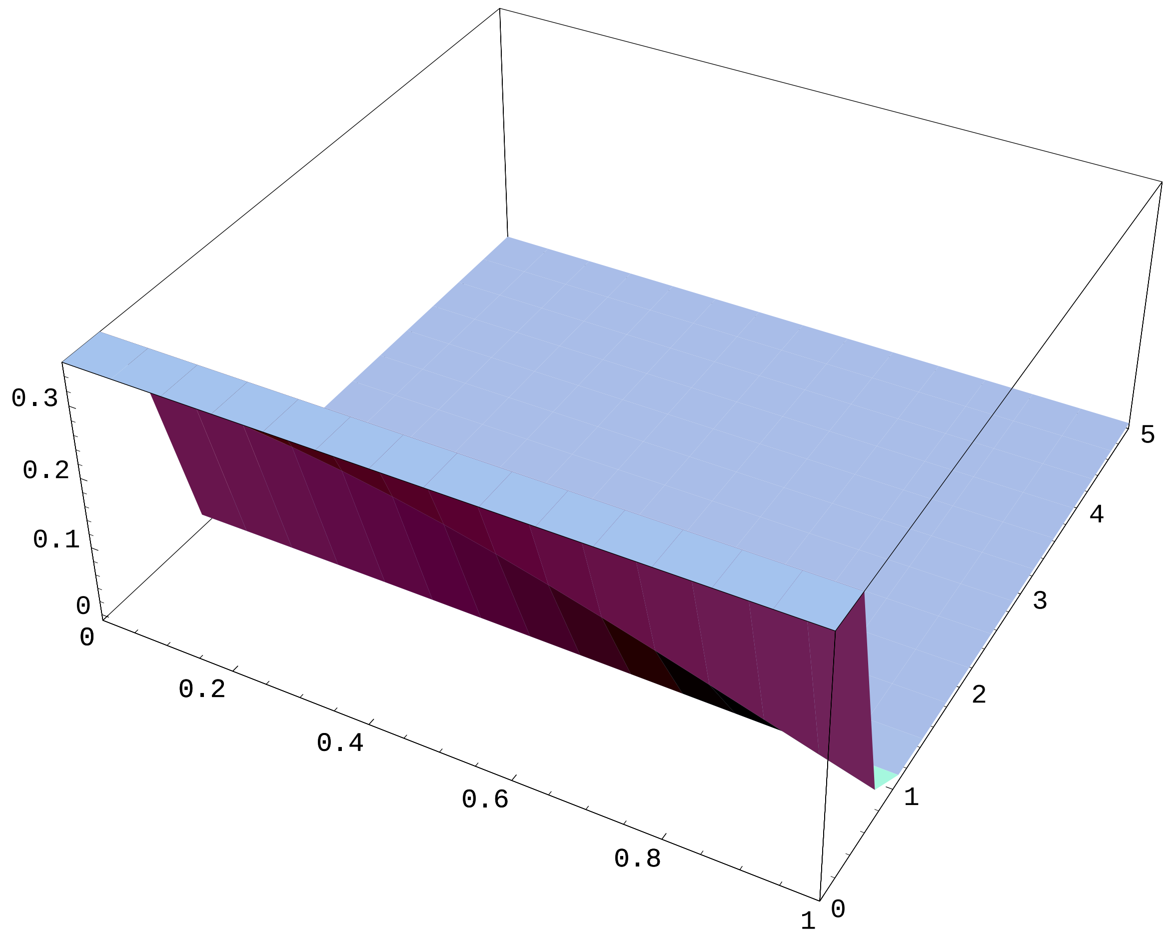
— *General::stop :*
Further output of Part::pspec will be suppressed during this calculation.

```
Temperatura = Sum[Ai * BesselJ[0, Sqrt[Lamda[[n]]] * r] * e^(-2 Lamda[[n]] * t), {n, 1, 30}]
```

$$\textbf{Temperatura} = \sum_{n=1}^{30} \textbf{Ai} * \textbf{BesselJ}\left[0, \sqrt{\textbf{Lamda[[n]]}} * \textbf{r}\right] * e^{-2\,\textbf{Lamda[[n]]} * \textbf{t}}$$

```
155.258 E^(-9.0622 t) BesselJ[0, 2.12864 r] -
 91.6299 E^(-48.7752 t) BesselJ[0, 4.93838 r] +
 61.8942 E^(-123.131 t) BesselJ[0, 7.84636 r] +
 61.8942 E^(-234.45 t) BesselJ[0, 10.8271 r] -
 44.3021 E^(-384.013 t) BesselJ[0, 13.8566 r] +
 33.1616 E^(-572.429 t) BesselJ[0, 16.9179 r] +
 33.1616 E^(-799.99 t) BesselJ[0, 19.9999 r] -
 25.7661 E^(-1066.84 t) BesselJ[0, 23.0959 r] -
 25.7661 E^(-1373.05 t) BesselJ[0, 26.2017 r] +
 20.6445 E^(-1718.67 t) BesselJ[0, 29.3144 r] -
 16.9621 E^(-2103.72 t) BesselJ[0, 32.4324 r] -
 16.9621 E^(-2528.22 t) BesselJ[0, 35.5543 r] +
 14.227 E^(-2992.17 t) BesselJ[0, 38.6793 r] -
 12.1383 E^(-3495.58 t) BesselJ[0, 41.8066 r] -
 12.1383 E^(-4038.46 t) BesselJ[0, 44.9358 r] +
 10.505 E^(-4620.81 t) BesselJ[0, 48.0667 r] -
 9.20165 E^(-5242.63 t) BesselJ[0, 51.1988 r] -
 9.20165 E^(-5903.92 t) BesselJ[0, 54.3319 r] +
 8.14334 E^(-6604.69 t) BesselJ[0, 57.466 r] +
 8.14334 E^(-7344.93 t) BesselJ[0, 60.6009 r] -
 7.27089 E^(-8124.64 t) BesselJ[0, 63.7363 r] +
 6.54212 E^(-8943.84 t) BesselJ[0, 66.8724 r] +
 6.54212 E^(-9802.5 t) BesselJ[0, 70.0089 r] -
 5.92628 E^(-10700.6 t) BesselJ[0, 73.1459 r] +
 5.40051 E^(-11638.3 t) BesselJ[0, 76.2833 r] +
 5.40051 E^(-12615.4 t) BesselJ[0, 79.4209 r] -
 4.94754 E^(-13631.9 t) BesselJ[0, 82.5589 r] +
 4.5541 E^(-14688. t) BesselJ[0, 85.6972 r] +
 4.5541 E^(-15783.5 t) BesselJ[0, 88.8356 r] -
 4.20985 E^(-16918.5 t) BesselJ[0, 91.9743 r]
```

```
Plot3D[Temperatura, {r, 0, 1}, {t, 0, 5}, Mesh → False]
```



- SurfaceGraphics -